Interference and coherence

What is called interference?

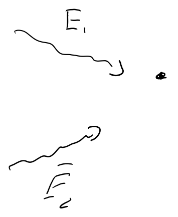
This effect can be observed for waves of very different nature. Usually, by interference one assume rather broad field of effect, when overlap of different waves do not simply result in their summation – in some places their will be maximas and in some places waves will cancel each other.

$$I \neq \sum_i I_i \quad (I \neq I_1 + I_2)$$

Demonstration

Video of interference of waves on water surface.

Let's try to sum up two optical waves $\bar{E}_1$ and $\bar{E}_2$. We assume they are polarized in random planes.



$$\bar{E}_p = \bar{E}_1 + \bar{E}_2 \quad \text{(superposition principle)}$$

Since we need intensity, then:

$$\langle \bar{E}_p^2 \rangle = \langle (\bar{E}_1 + \bar{E}_2)^2 \rangle = E_1^2 + E_2^2 + \underbrace{2\langle \bar{E}_1 \bar{E}_2 \rangle}_{\text{interference term}}$$

It is clear now, if $\bar{E}_1 \perp \bar{E}_2$, interference term will be 0, hence no interference will be observed. It will be simple summation of intensities based on superposition principle.

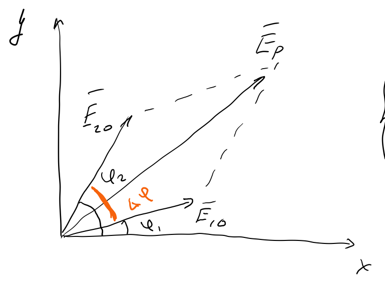
$$I_p = I_1 + I_2$$

From now on, we will only consider the cases when $\bar{E}_1 \parallel \bar{E}_2$.

Let's take two waves:

$$\begin{cases} E_1 = E_{10} \cos(\omega t - \bar{k}_1 \bar{r} + \delta_1) \\ E_2 = E_{20} \cos(\omega t - \bar{k}_2 \bar{r} + \delta_2) \end{cases}$$

If we sum them up:



$$E_p^2 = E_{10}^2 + E_{20}^2 + 2E_{10}E_{20}\cos\Delta\varphi$$

$$\begin{cases} \varphi_1 = \bar{k}_1 \bar{r} - \delta_1 \\ \varphi_2 = \bar{k}_2 \bar{r} - \delta_2 \end{cases}$$

$$\Delta\varphi = (\bar{k}_2 - \bar{k}_1)\bar{r} - (\delta_1 - \delta_2)$$

For intensities:

$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\bar{k}\bar{r})$$

$$k = \bar{k}_2 - \bar{k}_1 \quad \delta_1 = \delta_2 \; ! \; \text{initial phases are equal}$$

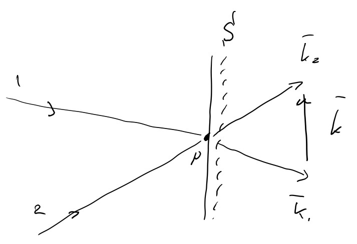
From here it is clear, that intensity at point p will remain constant as long as $\cos(\bar{k}\bar{r})$ remains constant.

Here we came to mathematical description of coherence – waves that can keep phase difference as constant are called coherent.

$$\boxed{\cos(\bar{k}\bar{r}) = \text{const}}$$

That also means that to observe stable interference pattern, one need coherent waves.

Let's see what one would observe on screen, if two plane waves will interfere on it:



$$\bar{k} = \bar{k}_2 - \bar{k}_1$$

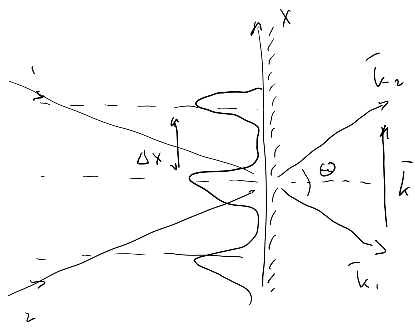
Please note, $S \parallel \bar{k}$

Now we need to answer here the question, what would be the surfaces of equal phase?

$\bar{k}\bar{r} = \text{const}$ – surface, that is $\perp \bar{k}$

This is geometrical ensemble of points,

where $I = \text{const}$.



That means on the screen we will have bright and dark lines – interference pattern.

In this coordinate system $\bar{k}\bar{r} = kx$.
What is the width of this interference lines?

Δx: $k\Delta x = 2\pi$ —> from one max to another.
1 period of cos.

$$|\bar{k}_1| = |\bar{k}_2| = \frac{2\pi}{\lambda}$$

$$k = \frac{2\pi}{\lambda}\sin\frac{\theta}{2}\cdot 2 \Rightarrow \boxed{\Delta x = \frac{2\pi}{k} = \frac{\lambda}{\theta}}$$

for small θ.

It is important to note, that there will be no interference if θ will be big and this is very simple to show.

Let's estimate angle θ so $\Delta x = 1\,mm$. This will clearly show that θ needs to be tiny.

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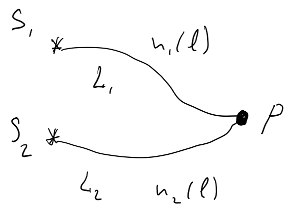
We will try to answer two important questions:

1) How we can calculate interference pattern?
2) What phase the waves will come with at point P?

Let's take two sources of light.



From S_1 the wave will go one way L_1, from S_2 it will go L_2.

Refractive index of $n_1(l)$ and $n_2(l)$ even can depend on coordinate.

In this case:

$$E_1 = E_{10}\cos(\omega(t - t_1))$$

$$t_1 = \int_{L_1}\frac{dl}{v_1} = \frac{1}{c}\underbrace{\int n_1(l)\,dl}_{\text{optical path } \delta_1}$$

$$t_1 = \frac{1}{c}\delta_1$$

Then phase difference:

$$\Delta\varphi = \frac{\omega}{c}\delta_2 - \frac{\omega}{c}\delta_1 = k_0\Delta$$

$$k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda_0}$$

Δ is optical path difference

$$\Delta = \int_{L_2} n_2(l)\,dl - \int_{L_1} n_1(l)\,dl$$

In special case:

$$n_1 = n_2 = n \Rightarrow \Delta = n(L_2 - L_1)$$

Finally, we can write:

$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi$$

where $\Delta\varphi = k_0\Delta$

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Let's take special case, when

$$I_1 = I_2 = I_0 \Rightarrow \boxed{I_p = 2I_0(1 + \cos k_0\Delta)}$$

Now we can derive conditions to observe max and min:

① max

$$k_0\Delta = 2\pi m$$

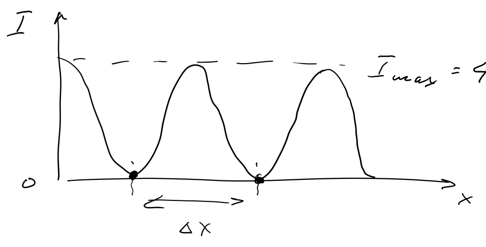
$$\Delta_{max} = 2m\frac{\lambda_0}{2}$$

② min

$$k_0\Delta = \pi + 2\pi m$$

$$\Delta_{min} = (2m+1)\frac{\lambda_0}{2}$$

If $I_1 = I_2 = I_0 \Rightarrow I_p = 2I_0(1 + \cos k_0\Delta)$.



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Demonstration.

Let's see such interference pattern.

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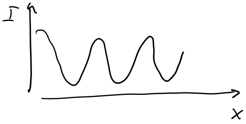
Visibility of interference pattern by Michelson

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$V = \frac{\sqrt{I_1 I_2}}{\left(\frac{I_1 + I_2}{2}\right)} \le 1$$

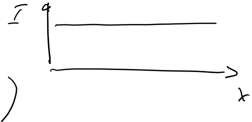
This is in fact ratio of geometric mean to arithmetic mean.

If $I_1 = I_2$ $V = 1$



If $I_1 >> I_2$ $V = 0$

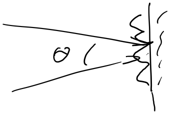
$(I_{max} = I_{min})$



That means:

1) For good interference we need to use beams of equal intensity.

2) θ should be small.



$$\Delta x = \frac{\lambda}{\theta}$$